Non-equilibrium dynamics of polymers and interfaces in random media: conjecture ψ = $d_S/2$ for the barrier exponent

Non-equilibrium dynamics of polymers and interfaces in random media: conjecture $\psi = d_s/2$ for the barrier exponent

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Abstract

We consider various random models (directed polymer, random ferromagnets, spin glasses) in their disorder-dominated phases, where the free-energy cost $F(L)$ of an excitation of length L presents fluctuations that grow as a power-law $\Delta F(L) \sim L^\theta$ with the 'droplet' exponent θ. Within the droplet theory, the energy and entropy of such excitations present fluctuations that grow as $\Delta E(L) \sim \Delta S(L) \sim L^{d_s/2}$ where d_s is the dimension of the surface of the excitation. These systems usually present a positive 'chaos' exponent $\zeta = d_s/2 - \theta > 0$, meaning that the free-energy fluctuation of order L^θ is a near cancellation of much bigger energy and entropy fluctuations of order $L^{d_s/2}$. Within the standard droplet theory, the dynamics is characterized by a barrier exponent ψ satisfying the bounds $\theta \leqslant \psi \leqslant d-1$. In this paper, we argue that a natural value for this barrier exponent is $\psi = d_s/2$: (i) for the directed polymer where $d_s = 1$, this corresponds to $\psi = 1/2$ in all dimensions; (ii) for disordered ferromagnets where $d_s = d-1$, this corresponds to $\psi = (d-1)/2$; (iii) for spin glasses where interfaces have a non-trivial dimension d_s known numerically, our conjecture $\psi = d_s/2$ gives numerical predictions in $d = 2$ and $d = 3$. We compare these values with the available numerical results for each case, in particular with the measure $\psi \simeq 0.49$ of Kolton–Rosso–Giamarchi (Kolton *et al* 2005 *Phys. Rev. Lett.* **95** 180604) for the non-equilibrium dynamics of a directed elastic string.

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1. Introduction

The non-equilibrium dynamics of disordered systems display a lot of striking properties such as ageing, rejuvenation, memory that have been studied both experimentally and theoretically (see [1] and references therein). There is now a growing consensus that the understanding of

these effects in finite-dimensional random systems requires the introduction of some coherence length $L_T(t)$, with the following meaning. If the thermal dynamics at temperature T starts at $t = 0$ from a random initial condition, the coherence length $L_T(t)$ associated with time t separates the smaller lengths $l < L_T(t)$ which are quasi-equilibrated from the bigger lengths $l > L_T(t)$ which are completely out of equilibrium. The two-time ageing properties and the rejuvenation and memory effects for more complicated temperature-cycle experiments can be then understood in terms of this coherence length $L_T(t)$ [1]. However, what has remained very controversial over the years is whether the coherence length $L_T(t)$ grows algebraically or logarithmically with time or equivalently whether there exists a positive barrier exponent ψ as we now recall.

1.1. Logarithmic dynamics with some barrier exponent $\psi > 0$

The activated nature of the non-equilibrium dynamics is natural within the droplet scaling theory proposed both for spin glasses [2–4] and for the directed polymer in a random medium [5] (see section 2 for more details). Barriers are then expected to grow as a power-law at large scale

$$B(L) \sim L^\psi \tag{1}$$

where the exponent $\psi > 0$ is constant in the whole low-temperature phase $T < T_c$ and is a property of the large-scale zero-temperature fixed point. The typical time $t_{\text{typ}}(L)$ associated with scale L grows as an exponential

$$\ln t_{\text{typ}}(L) \sim B(L) \sim L^\psi. \tag{2}$$

As a consequence, the non-equilibrium dynamics starting at time $t = 0$ is expected to involve only logarithmic functions of time via the characteristic length-scale $L(t)$ associated with time t

$$L(t) \sim (\ln t)^{\frac{1}{\psi}}. \tag{3}$$

A simple one-particle one-dimensional disordered model where this type of activated dynamics occurs is the Sinai model [6] where one particle diffuses in a random Brownian potential of exponent $\psi = 1/2$: the diffusion is then logarithmic with $L(t) \sim (\ln t)^2$. One-time and two-time properties of the non-equilibrium dynamics can be computed at large time via a strong-disorder renormalization procedure that yields asymptotic exact results at large times [7].

1.2. Algebraic dynamics with some dynamical exponent z

In the alternative scenario of 'algebraic' dynamics, barriers grow at most logarithmically with L, i.e. the exponent ψ of equation (1) vanishes $\psi = 0$. Time scales and length scales are then related by some dynamical exponent z

$$t_{\text{typ}}(L) \sim L^z \tag{4}$$

$$L(t) \sim t^{\frac{1}{z}} \tag{5}$$

instead of the logarithmic relations of equations (2) and (3). As a consequence, the ageing properties of the non-equilibrium dynamics involve ratios of times (instead of ratios of logarithms of the times).

A simple one-particle one-dimensional disordered model where this type of algebraic ageing occurs is the Bouchaud trap model [8] (see [9] for the mean-field version).

1.3. Debate on the phase-space structure at large scales

This question on the value of the barrier exponent ψ amounts more generally to understand the phase-space structure of polymers or interfaces in random media. Within the droplet theory [2, 3, 5] (see section 2), static properties are governed by low-energy excitations characterized by the 'droplet' exponent θ and by the dimensionality d_s of the surface of the excitations, whereas the dynamics involves *a priori* another exponent ψ satisfying the bounds $\theta \leqslant \psi \leqslant d - 1$ [3–5]. Whereas the exponents θ and d_s for the statics are known either exactly or numerically in various models, the numerical measure of the barrier exponent ψ turns out to be much more difficult from the point of view of computational complexity [10]. As a consequence, the value of ψ has remain very uncertain even numerically in many random models. The aim of this paper is to explain that the value $\psi = d_s/2$ for the barrier exponent is very natural for disordered models that have a positive chaos exponent $\zeta = d_s/2 - \theta > 0$ and to compare with available numerical data.

1.4. Organization of the paper

In section 2, we recall the essential properties of the droplet scaling theory and present our general arguments for the conjecture $\psi = d_s/2$. We then discuss this conjecture with a comparison to existing numerical results for the following models: directed polymers in random media (section 3), disordered ferromagnets (section 4) and spin glasses (section 5). Section 6 contains our conclusions.

2. Arguments in favor of the value $\psi = d_s/2$ for the barrier exponent

In this section, before explaining the conjecture $\psi = d_s/2$ for the barrier exponent, we need to recall the main statements of the droplet scaling theory proposed both for spin glasses [2–4] and for the directed polymer in a random medium [5].

2.1. Reminder on equilibrium properties within the droplet theory

2.1.1. Statistics of excitations above the ground state. At very low temperature $T \to 0$, all observables are governed by the statistics of low-energy excitations above the ground state. An excitation of large length l costs a random energy

$$E_{\text{exc}}(l, T = 0) \sim l^{\theta} u \tag{6}$$

where θ is the so-called droplet exponent [2–5], and where u is a positive random variable distributed with some law $Q_0(u)$ having some finite density at the origin $Q_0(u = 0) > 0$. A low-temperature disorder-dominated phase exists whenever the droplet exponent θ is positive $\theta > 0$.

From equation (6), the probability distribution of large excitations $l \gg 1$ reads within the droplet theory

$$dl \rho_{\text{exc}}(l) \sim \frac{dl}{l} \, e^{-\beta E_{\text{exc}}(l, T=0)} \sim \frac{dl}{l} \, e^{-\beta l^{\theta} u} \tag{7}$$

where the factor dl/l comes from the notion of independent excitations [3]. In particular, its average over the disorder follows the power law

$$dl \overline{\rho_{\text{exc}}(l)} \sim \int_0^{+\infty} du \, Q_0(u) \frac{dl}{l} \, e^{-\beta l^{\theta} u} = T Q(0) \frac{dl}{l^{1+\theta}}. \tag{8}$$

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Since correlation functions at large distance are directly related to the probability of large excitations, the low-temperature phase is very non-trivial from the point of view of correlations lengths: the typical exponential decay of equation (7) indicates a finite typical correlation length $\xi_{\mathrm{typ}}(T)$, whereas the averaged power-law behavior of equation (8) means that the averaged correlation length $\xi_{av}(T)$ is actually infinite in the whole low-temperature phase $\xi_{av}(0 < T \leqslant T_c) = \infty$.

2.1.2. Low-temperature phase governed by a zero-temperature fixed point. According to the droplet theory, the whole low-temperature phase $0 < T < T_c$ is governed by a zero-temperature fixed point. However, many subtleties arise because the temperature is actually 'dangerously irrelevant'. The main conclusions of the droplet analysis [3, 5] can be summarized as follows. The scaling of equation (6) governs the free energy cost of an excitation of length l, provided one introduces the typical correlation length $\xi_{\mathrm{typ}}(T)$ to rescale the length l

$$F_{\mathrm{exc}}(l, 0 < T < T_c) = \left(\frac{l}{\xi_{\mathrm{typ}}(T)} \right)^{\theta} u. \tag{9}$$

Here as before, u denotes a positive random variable distributed with some law $Q(u)$ having some finite density at the origin $Q(u = 0) > 0$. But this droplet free energy turns out to be a near cancellation of much larger energy and entropy contributions that scale for large l as [3, 5]

$$
\begin{aligned}
E_{\mathrm{exc}}(l, 0 < T < T_c) &\sim \sigma(T) l^{\frac{d_s}{2}} w + e_1(T) l^{\theta} \\
T S_{\mathrm{exc}}(l, 0 < T < T_c) &\sim \sigma(T) l^{\frac{d_s}{2}} w + \cdots
\end{aligned}
\tag{10}
$$

where d_s represents the dimension of the surface of the excitation. The random variable w of order $O(1)$ and of zero mean is expected to be Gaussian distributed. The argument is that the energy and entropy are dominated by small-scale contributions of random sign [3, 5], whereas the free energy is optimized on the coarse-grained scale $\xi_{\mathrm{typ}}(T)$. A very important consequence of the difference in scaling of the free-energy fluctuation of equation (9) and of the energy–entropy fluctuations of equation (10) is the presence of disorder and temperature chaos in the whole low-temperature phase with the so-called chaos exponent [3, 5, 11]:

$$\zeta = \frac{d_s}{2} - \theta > 0. \tag{11}$$

Note that, numerically, temperature chaos is usually harder to observe than disorder chaos (see [12, 13] and references therein).

For numerical simulations, it is important to stress that the term of order $l^{ds/2}$ in equation (10) is the leading term for large l, but that there exists a subleading term of order l^{θ} to recover the free-energy fluctuations of equation (9). And since the amplitude $\sigma(T)$ of the leading term vanishes in the limit of $T = 0$ (as $\sigma(T) \propto T \times T^{1/2} = T^{3/2}$ [12]), whereas the amplitude $e_1(T)$ of the subleading term in the energy remains finite $e_1(T = 0) > 0$ (equation (6)), one needs to simulate sufficiently large excitations to reach the size l where the leading term of equation (10) becomes much bigger than the subleading term:

$$\sigma(T) l^{\frac{d_s}{2}} \gg e_1(T) l^{\theta}. \tag{12}$$

2.2. Reminder on the non-equilibrium dynamical properties within the droplet theory

Within the standard droplet theory [2–4], the non-equilibrium dynamics is governed by large-scale barriers $B(L) \sim L^{\psi}$ where the barrier exponent ψ satisfies the bounds

$$\theta \leqslant \psi \leqslant d - 1. \tag{13}$$

The lower bound comes from the fact that the barrier $B(L)$ to create a droplet excitation of size L cannot be less than the free-energy cost of the droplet (equation (9)). The upper bound comes from the expectation that the barrier cannot have a greater exponent than the barrier L^{d-1} needed to create a non-optimized excitation of surface L^{d-1}. In particular, whenever there exists a low-temperature disorder-dominated phase with a positive droplet exponent $\theta > 0$, the barrier exponent is strictly positive $\psi \geqslant \theta > 0$ and leads to some logarithmic dynamics (see equation (3)). Since in this paper we focus on the value of the barrier exponent ψ, we refer the reader to [3, 4, 14] for a detailed description of other properties of the droplet dynamics.

2.3. Arguments in favor of the value $\psi = d_s/2$

From the point of view of the dynamical exponent ψ, there has been a long-standing difference between

(i) the directed polymer model, where the assumption that the barrier exponent coincides with the droplet exponent has been made from the very first article that has introduced the model [15] (see section 3 for more details)

$$\text{usual assumption for the directed polymer}: \psi = \theta; \tag{14}$$

(ii) spin glasses, where it has been quickly clear that the barrier exponent in strictly bigger than the droplet exponent (see section 5 for more details), because they are distinct below the lower critical dimension. In dimension $d = 1$, the exact solution [2] yields

$$\text{1D spin glass}: \psi = 0 > \theta = -1 \tag{15}$$

and in dimension $d = 2$ these two exponents do not have the same sign

$$\text{2D spin glass}: \psi > 0 > \theta. \tag{16}$$

The usual explanation of this difference between the two models (i) and (ii) is that the directed polymer case would be much more 'simple' than the spin-glass case, that its phase space would be characterized by a single exponent θ, whereas in spin glasses the barrier scaling is not related to the scaling of the free-energy minima. In this paper, we propose another scenario, based on the observation that in any disorder system presenting a positive chaos exponent $\zeta = d_s/2 - \theta > 0$ (and in particular for the directed polymer), the description of the phase space requires at least two exponents which are the droplet exponent θ for free-energy fluctuations (equation (9)) and the exponent $d_s/2$ that governs energy and entropy fluctuations (equation (10)). The assumption of equation (14) is then equivalent to the very strong requirement that the global free-energy optimization of order L^θ that results from a near cancellation of much bigger energy and entropy random contributions of order $L^{ds/2}$ is satisfied *all along the dynamical trajectories*. The alternative scenario that we propose in this paper is that for any dynamics containing only local moves of the polymer or interface, the barrier exponent ψ is equal to the energy–entropy fluctuation exponent $d_s/2$ (equation (10))

$$\psi = \frac{d_s}{2}. \tag{17}$$

The physical interpretation is that the equality $\psi = \theta$ would be possible only via a non-local dynamics that would allow a global reorganization of the polymer or interface at each time step, whereas any local dynamics will see barriers that are dominated by small-scale contributions of random sign.

In the remaining sections, we discuss this conjecture for various disordered models and compare with the available numerical results.

3. Non-equilibrium dynamics of directed polymers in random media

3.1. Reminder on the statics

The directed polymer in a random medium (see [16] for a review) is a model where the various statements of the droplet scaling theory have been successfully tested. The exponent θ of equation (6) is exactly known in one dimension $\theta(d = 1) = 1/3$ [17–19] and for the mean-field version on the Cayley tree $\theta(d = \infty) = 0$ [20]. In finite dimensions $d = 2, 3, 4, 5, \ldots$, the exponent $\theta(d)$ has been numerically measured, with values of order $\theta(d = 2) = 0.244$ and $\theta(d = 3) = 0.186$ [21, 22]. The statistics of equation (8) for the low-energy excitations as a function of their size l very well describes the numerical data in the regime $1 \ll l \ll L$ in dimensions $d = 1, 2, 3$ [23]. Finally, the scaling of equation (10) for the energy and entropy fluctuations have been numerically checked in various dimensions in [5, 24, 25]. Let us stress again that the difference between free-energy fluctuations of equation (9) and energy fluctuations of equation (10) can be seen only for sufficiently large scale L [5, 24, 25] (see the discussion before equation (12)).

3.2. Discussion of the conjecture $\psi = d_s/2 = 1/2$ for the barrier exponent

To the best of our knowledge, all papers discussing the non-equilibrium dynamics of the directed polymer seem to have assumed the equality of equation (14) between the barrier exponent ψ and the droplet exponent θ. This assumption was first made in the very first paper [15] introducing the directed polymer model, i.e. before the droplet analysis of the model [5]. More recently, many papers consider that the equality of equation (14) has been 'proven' in [26] up to possible logarithmic corrections. In our opinion, the arguments contained in [26] are problematic since the authors of [26] seem to be unaware of the crucial difference in scaling between free-energy fluctuations and energy fluctuations (equations (9) and (10)). For instance, they state that the dynamics is controlled by 'energy barriers', which have the 'same scaling as free-energy fluctuations', because it is a 'zero-temperature fixed point', but as recalled above, within the droplet theory, the properties of the 'zero-temperature fixed point' are instead the different scalings of equations (9) and (10).

We have explained above in section 2.3 our general arguments in favor of the value $\psi = d_s/2$. For the directed polymer of dimension $d_s = 1$ in a random medium of dimension $1 + d$, this corresponds to

$$\psi_{\mathrm{DP}} = \frac{d_s}{2} = \frac{1}{2}. \tag{18}$$

We now compare with the available numerical results.

3.3. Comparison with available numerical results on the non-equilibrium dynamics

Although ageing effects for the directed polymer have been fitted with algebraic time scalings by various authors [27], the more recent work of Kolton, Rosso and Giamarchi [28] shows that

(a) the growing length $L(t)$ cannot be fitted by a power-law $L(t) \sim t^{1/z}$ at large times, although the short time relaxation could be fitted with some effective exponent $z(T)$ that strongly depends on temperature. This could explain why the first numerical fits [27] see apparent algebraic ageing forms.

(b) the growing length $L(t)$ can be fitted with the logarithmic form $L(t) \sim (\ln t)^{1/\psi}$ at large times, and the value of the barrier exponent ψ is asymptotically size and time independent as it should.

(c) the value of ψ measured in [28] is $\psi \simeq 0.49$ in the last three decades. The interpretation of the authors of [28] that believe in the identity $\psi = \theta = 1/3$ is that barriers contain strong logarithmic corrections $B(L) \sim L^{1/3}(\ln L)^\mu$. Our interpretation is in contrast in that the measured value $\psi \sim 1/2$ is actually the correct one.

4. Non-equilibrium dynamics in disordered ferromagnets

4.1. Numerical results on coarsening in disordered ferromagnets

The non-equilibrium dynamics in pure ferromagnets in the low-temperature phase $T < T_c$ is well understood via the characterization of domain coarsening [29]. In the presence of quenched disorder however, the large time behavior of the characteristic length-scale $R(t)$ of the coarsening process has remained controversial between logarithmic behavior [15, 30, 31]

$$R(t) \sim (\ln t)^x \tag{19}$$

with a universal exponent x discussed below and power-law growth [32]

$$R(t) \sim t^{\frac{1}{z(T,\epsilon)}} \tag{20}$$

with an exponent $z(T, \epsilon)$ that depends both on the temperature T and on the disorder strength ϵ.

However, as stressed in the recent work [33], the numerical simulations of coarsening in disordered ferromagnets do not really reach the large-scale large-time regime, since the maximal size $R_{\max}$ measured is sometimes only of order $R_{\max} \sim 7$ in unit of lattice spacings at the end of the simulation (see for instance figure 4(a) of [32] or figure 1 of [33]). As a consequence, the available numerical simulations on coarsening with quenched disorder are not very conclusive for the asymptotic regime of $R(t)$.

4.2. Relation between the exponent x in $d = 2$ and the directed polymer barrier exponent ψ_{DP}

Since the directed polymer model discussed in the previous section has been precisely introduced as a model of domain wall in two-dimensional disordered ferromagnet [15], one expects some relation between the barrier exponent ψ_{DP} of the directed polymer in $1+1$ and the exponent x of equation (19) governing the domain growth in the two-dimensional disordered ferromagnet. The first possibility would be simply [31].

$$x_{\text{simple}} = \frac{1}{\psi_{DP}}, \tag{21}$$

meaning that the dynamics is governed by the barriers associated with the domain scale $R(t)$. However, Huse–Henley [15] have proposed another scenario leading to the higher value

$$x_{HH} = \frac{(2 - \zeta)}{\psi_{DP}} \tag{22}$$

where $\zeta = 2/3$ is the roughness exponent of the directed polymer in $1 + 1$. The argument leading to the value of equation (22) can be summarized as follows [15, 31, 29]. The relevant interfaces during the coarsening process are not directed polymers but curved polymers with a typical curvature radius of order $R(t)$ itself. It can be consider as directed up to the size l where the roughness l^ζ is of the same order of the curvature $l^2/R(t)$ yielding $l(t) \sim (R(t))^{1/(2-\zeta)}$. The barriers associated with these directed parts scale as $(l(t))^{\psi_{DP}} \sim (R(t))^{\psi_{DP}/(2-\zeta)}$ leading to equation (22).

Note that these arguments usually go along with the assumption $\psi_{DP} = \theta_{DP} = 1/3$ (see the previous section on the directed polymer) yielding the values $x_{\text{simple}} = 3$ and $x_{HH} = 4$ [15, 31, 29]. With the value $\psi_{DP} = 1/2$ of our conjecture discussed in the previous section for

the directed polymer, the values of the exponent x are respectively $x_{\text{simple}} = 2$ and $x_{\text{HH}} = 8/3$. In the following, we argue that within our analysis, it is the value $x_{\text{simple}} = 2$ which is natural for disordered ferromagnets in $d = 2$.

4.3. Conjecture $\psi = d_s/2 = (d - 1)/2$ for the barrier exponent

Within our analysis where the barrier exponent ψ is governed by the dimensionality d_s of the interface that determines the energy and entropy fluctuations (equation (10)), we expect that in disordered ferromagnets where $d_s = d - 1$ the barrier exponent is

$$\psi = \frac{d_s}{2} = \frac{d - 1}{2} \tag{23}$$

irrespectively of the directed or curved nature of the interface, since it is governed by small-scale contributions. In particular, in domain coarsening, we expect the 'simple' relation that generalizes equation (21)

$$x = \frac{1}{\psi} = \frac{2}{d - 1}. \tag{24}$$

Again, as explained above, numerical data on coarsening with disorder do not allow a precise measure of the exponent x because $R(t)$ of equation (19) is never very large in simulations (see [31] for a more detailed discussions of the results of various fits).

5. Non-equilibrium dynamics in spin glasses

Many numerical works have studied non-equilibrium properties in spin glasses. Here, again there is a controversy between logarithmic dynamics (see for instance [34, 35]) and algebraic dynamics (see for instance [36, 37]).

5.1. Discussion of the conjecture $\psi = d_s/2$ for spin glasses in $d = 2$

In dimension $d = 2$, there is no spin-glass phase because the droplet exponent θ is negative $\theta < 0$ so that $T_c = 0$. Nevertheless, it is interesting to measure the values of the droplet exponent θ and of the fractal dimension d_s of the surface of excitations above the ground state. Recent estimates are $\theta \simeq -0.287(4)$ (see [38] and references therein) and $d_s \simeq 1.274(2)$ (see [39] and references therein). Note that it has been recently argued that these interfaces are described by SLE evolutions implying some simple relation between θ and d_s [40].

Using $d_s \simeq 1.274$ [39], the present conjecture $\psi = d_s/2$ for the barrier exponent would corresponds to a numerical value of order

$$\text{2D spin glass}: \psi = \frac{d_s}{2} \simeq 0.637. \tag{25}$$

This value is very close to the estimate $\psi \simeq 0.61$ obtained via Monte Carlo simulations [36]. On the other hand, Amoruso, Hartmann and Moore [41] have tried to measure the barrier exponent ψ of the highest barrier of systems of sizes $L \leqslant 40$ yielding the numerical bounds $0.25 < \psi < 0.54$. The uncertainty shows the difficulty of the numerical measure of ψ so that their upper bound does not seem to us sharp enough to rule out the value of equation (25).

5.2. Discussion of the conjecture $\psi = d_s/2$ for spin glasses in $d = 3$

In dimension $d = 3$, the droplet exponent θ is positive $\theta > 0$ so there exists a spin-glass phase with $T_c > 0$. Recent estimates for the droplet exponent θ are of order $\theta \simeq 0.19(2)$ [42],

$\theta \simeq 0.27$ [43], $\theta \simeq 0.24(1)$ [44]. The fractal dimension d_s of the surface of excitations above the ground state is of order $d_s \simeq 2.6$ (see [45] and references therein). Using the latter, the present conjecture $\psi = d_s/2$ for the barrier exponent would correspond to a numerical value of order

$$\text{3D spin glass} : \psi = \frac{d_s}{2} \simeq 1.3. \tag{26}$$

This has to be compared with the value $\psi \sim 1.0$ estimated by Berthier and Bouchaud from their ageing simulations [35]. Again, the precision of this numerical estimate does not seem sufficient to rule out the value of equation (26). We refer the reader to [35] for the experimental values of the exponent ψ reported in the literature that varies between 0.3 and 1.9 [35].

6. Conclusion

In this paper, we have proposed that, in disordered systems characterized by a positive chaos exponent $\zeta = d_s/2 - \theta > 0$, the large time dynamics is governed by the barrier exponent $\psi = d_s/2$. We have explained why this value $\psi = d_s/2$ is natural within the droplet scaling picture, where the exponent $d_s/2$ governs the energy–entropy fluctuations (equation (10)) and is greater than the droplet exponent θ of free-energy fluctuations (equation (9)). We have then discussed our conjecture for the following models:

(i) for the directed polymer, where $d_s = 1$, our conjecture gives $\psi = 1/2$ in all dimensions;
(ii) for disordered ferromagnets, where $d_s = d - 1$, our conjecture corresponds to $\psi = (d - 1)/2$;
(iii) for spin glasses where interfaces have a non-trivial fractal dimension d_s known numerically, our conjecture $\psi = d_s/2$ gives numerical predictions in $d = 2$ and $d = 3$.

In each case, we have compared with the available numerical data on ψ, in particular with the work of Kolton–Rosso–Giamarchi[28] who have measured the barrier exponent $\psi \simeq 0.49$ for the non-equilibrium dynamics of a directed elastic string. For disorder spin models, either disordered ferromagnets or spin glasses, the available numerical estimates of ψ are not sufficiently precise to support or exclude our conjecture.

If the conjecture $\psi = d_s/2$ is correct, this means that the numerical measure of this barrier exponent in dynamical simulations requires to study samples of sizes L sufficiently large, where in the corresponding statics the free-energy fluctuations and the energy–entropy fluctuations have reached their asymptotic regimes of equations (10) and (9), i.e. one needs to be in the regime of equation (12) for the statics. We hope that this explicit static criterion will help to identify the regime where dynamic simulations are likely to measure the asymptotic barrier exponent ψ relevant at large scales.

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